

## FRACTAL DIMENSION AND CAUSALITY RELATIONSHIPS WITH FOLIAR PARAMETERS: CASE STUDY AT *ALNUS GLUTINOSA* (L.) GAERTN.

Florin SALA<sup>1,\*</sup>, Adina-Daniela DATCU<sup>1,2</sup>, Ciprian RUJESCU<sup>3</sup>

<sup>1</sup>Soil Science and Plant Nutrition; <sup>3</sup>Mathematic and Statistics; Banat University of Agricultural Sciences and Veterinary Medicine "King Michael I of Romania" from Timisoara, 300645, Romania

<sup>2</sup>Biology-Chemistry Department; West University of Timișoara, Romania

\*Corresponding author's e-mail: florin\_sala@yahoo.com

Received 1 April 2020; accepted 13 May 2020

### ABSTRACT

This paper purpose was to analyze leaves shape geometry through fractal dimension parameter for *Alnus glutinosa* (L.) Gaertn, a species that can be found generally in alluvial forests, situated below 1000 m, but growing sporadically up to 1800 m. This index, which can be determined quickly and cheaply, is appropriate in biomonitoring studies in urban or non-urban environments, because this species is also often found in parks, in towns and cities. Fractal dimension can be analyzed in relation with various morphometric parameters, as perimeter, scanned leaf area, leaf length and width. In this study, mathematical models between these indices and fractal dimension were developed, statistical safety being assured. Fractal dimension  $D$  can be a stable index for the leaves geometry characterization, having a low degree of variability when compared with  $L$ ,  $W$ ,  $Per$  or  $SLA$ , which presented a high degree of variability, noticed based on variation coefficient  $CV$ .

**KEY WORDS:** *Alnus glutinosa*, fractal analysis, leaf parameters, isoquants, RMSE

### INTRODUCTION

*Alnus glutinosa* (L.) Gaertn., black alder, is a relatively small, broad-leaved tree, native to almost all Europe (Ellenberg, 2009), being one of the three *Alnus* species widespread in this region (Aniszewska *et al.*, 2019). It has simple, dark green leaves, which morphologically are obovate (Mitchell, 1974). This species grows below 1000m in elevation, although in the mountains of central Europe it can occasionally be found along watercourses up to 1800m (Houston Durrant *et al.*, 2016). Common alder establishes populations in various environments, due to the fact that is adapted to a wide range of temperatures and is quite frost-tolerant (Rutkowski *et al.*, 2019). Although black alder is not really common, it is an important component in open landscapes, especially along river banks and in marshy areas (Rutkowski *et al.*, 2019). For this species, researches were realized regarding wood structure (Yaman, 2009; Han *et al.* 2015), growth patterns (Vacek *et al.*, 2016; Socha & Ochał, 2017), wood quality (Kaliniewicz *et al.*, 2018), weight, size, and storage of seeds (Gosling *et al.*, 2009; Kaliniewicz & Trojanowski, 2011)

Plant functional characteristics reflect plant adaptations to different environments and utilization of resources, and therefore provide important information for the analyses of community assembly (McGill *et al.*, 2006; Ackerly & Cornwell, 2007) and ecosystem functioning and services (Díaz *et al.*, 2007). Moreover, studies regarding the relation between leaves features (degree of lobing, leaf apex, size and outline shape, base morphology and presence or absence of marginal teeth) and climate (average annual temperature, precipitation) over the last decades has revealed that, for many plant species, physical characteristics vary predictably in response to local environmental variation (MacLeod & Steart, 2015).

Fractal dimension analysis for leaves was developed starting with 1977 by Mandelbrot (1977) and there are different types of methods generally used like box-counting method (Tricot, 1995; Buczkowski *et al.*, 1998) or, for multifractal dimension Minkowski's method (Costa & Cesar, 2000).

The study purpose was the analyze and characterization of leaves geometry through fractal analysis and estimation of determination relations between foliar parameters and fractal dimensions (D) for *Alnus glutinosa* (L.) Gaertn.

#### **MATERIAL AND METHOD**

The study evaluated fractal geometry of black alder leaves and the causality relation between fractal dimension and foliar parameters.

The biological material was represented by *Alnus glutinosa* (L.) Gaertn samples. Leaf probes were randomized collected and contained the natural size variation of leaves. 100 leaves with variable sizes were investigated. The black alder leaves are ovate or roundish, with an obtuse apex, an acuminate base and margins coarsely and doubly serrate figure 1. Leaves images were obtained through 1:1 scanning. Some foliar parameters were studied: leaf length (L), leaf width (W), perimeter (Per), scanned leaf area (SLA) (Sala *et al.*, 2015).



**FIGURE. 1.** *Alnus glutinosa* (L.) Gaertn. leaf

For the fractal analysis, box-counting method was used (Voss, 1985). It was applied on binarized images (Rasband, 1997). Fractal dimensions, intermediary values, relation (1) and mean value, relation (3) were obtained in statistical safety conditions ( $R^2$  for D had values from 0.997 to 0.998).

$$D = m \left[ \frac{\ln(F)}{\ln \varepsilon} \right] \quad (1)$$

where: D – fractal dimension;  
 m – slope to regression line, from equation (2);  
 F – number of new part;  
 $\varepsilon$  – scale applied to an object.

$$m = (n \sum SC - \sum S \sum C) / (n \sum S^2 - (\sum S)^2) \quad (2)$$

where: m – slope of the regression line;  
 S – log of scale or size;  
 C – log of count;  
 n – number of size;

$$Mean D = \sum(D) / GRIDS \quad (3)$$

For experimental data analysis, ANOVA single factor test was utilized for the variance evaluation and the safety of experimental data.

Causality relations between D and foliar parameters was studied through multiple and simple regression analysis and as statistical safety parameters regression coefficient ( $R^2$ ), p parameter, RMSE, relation (4) and SEP relation (5) were used. For imagistic analysis of leaf probes and experimental data, ImageJ, EXCEL and PAST (Hammer *et al.*, 2001) were used.

$$RMSE = \sqrt{\frac{1}{n} \sum_{j=1}^n (y_j - \hat{y}_j)^2} \quad (4)$$

$$SEP = \sqrt{p(1-p) \left( \frac{1}{n_1} + \frac{1}{n_2} \right)} \quad (5)$$

## RESULTS AND DISCUSSIONS

The determinations and measurements of black alder leaves conducted to data obtained regarding leaves lengths (L) and widths (W). On the scanned images in a 1:1 ratio, scanned leaf area, (SLA) and perimeter (P) were determined. Considering leaves

dimensions, 7 size groups were obtained, and the studied parameters varied: L=4.10±0.08 cm (G1) to L=10.15±0.13 cm (G7); W=3.57±0.12 cm (G1) to W=9.37±0.26 cm (G7), SLA=10.98±0.64 cm<sup>2</sup> (G1) to SLA=69.01±2.99cm<sup>2</sup> (G7), Per=134.72±3.79 mm (G1) to Per=340.58±7.35 mm (G7).

Fractal analysis, box-counting, conducted to D values with a high statistical safety (R<sup>2</sup>=0.997 to R<sup>2</sup>=0.998), with a variation between D=1.737±0.003 (G1) and D=1.811±0.004 (G7). Foliar parameters and fractal dimensions (D) are presented in table 1.

The analysis of the values for the determined biometric parameters (L, w, Per, SLA) and fractal values (D) conducted to a differentiated variability of common alder leaves, mathematic instrument being coefficient of variation (CV). Thus, the coefficient of variation had the value CV<sub>L</sub>=28.8263 for leaves length, CV<sub>W</sub>=30.3299 for leaves width, CV<sub>Per</sub>=29.1355 for leaves perimeter, CV<sub>SLA</sub>=59.0168 for scanned leaf area SLA. These values presented a high degree of variability for leaves depending on the dimensional parameters.

**TABLE 1. Foliar parameters and fractal dimension (D) data for black alder *Alnus glutinosa* (L) Gaertn leaves (mean values depending on size groups)**

Sample	L	W	Per	SLA	D	R <sup>2</sup> for D
G1	4.10±0.08	3.57±0.12	134.72±3.79	10.98±0.64	1.737±0.003	0.997
G2	5.06±0.11	4.61±0.05	171.10±2.44	17.34±0.49	1.746±0.002	0.997
G3	5.97±0.10	5.37±0.08	195.90±1.88	22.82±0.28	1.757±0.003	0.997
G4	6.43±0.14	5.85±0.09	210.61±1.30	26.69±0.32	1.761±0.001	0.997
G5	6.79±0.11	6.17±0.11	231.95±1.50	32.27±0.52	1.768±0.003	0.997
G6	8.22±0.15	7.29±0.15	270.36±5.26	42.95±1.49	1.771±0.003	0.998
G7	10.15±0.13	9.37±0.26	340.58±7.35	69.01±2.99	1.811±0.004	0.998

L – leaf length; W – leaf width; Per – leaf perimeter; SLA – scanned leaf area; D – fractal dimension

**TABLE 2. ANOVA single factor**

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	3784941	4	946235.2	1040.329	1.7E-246	4.690517
Within Groups	472967.8	520	909.5534			
Total	4257908	524				

Alpha = 0.001

Regarding fractal dimensions (D), a low variability was noticed (CV<sub>D</sub> = 1.4254) when compared with the other studied parameters. This fact shows that the stability of fractal dimensions in leaves typology expression for the studied species, similar results being communicated also in other studies (Sala et al., 2017).

Also, a high independence degree of D values when describing a plant species was noticed, the specific patterns of leaves shapes varying in a short range in relation with them size. This shows fractal analysis capacity to describe and characterize

shapes fractal geometry, in this case, for black alder leaves shape.

Fractal analysis comprised the typology or the specific of fractal geometry of leaves shapes, generating D values with a high degree of stability. The highest variability in black alder leaves characterization was for SLA ( $CV_{SLA}=59.0168$ ) and the highest stability was obtained for fractal dimensions (D), the value of variation coefficient being  $CV_D=1.4254$ . Correlation analysis conducted to the data presented in table 3. After the completion of the analysis of the obtained values, very high and high positive correlations were noticed between the studied foliar parameters, SLA and the values of fractal dimensions D.

Regression analysis evaluated the contribution of foliar parameters (L, W, Per, SLA), as parameters which define black alder leaves geometry for obtaining fractal dimensions (D).

The D variation in relation with the studied parameters was described by relation (6), with  $R^2=0.835$ ,  $p \ll 0.001$ , and equations terms of the relation (6). After analyzing the coefficients values of relation (6), a differentiated contribution of leaf parameters, which defines in general, the fractal geometry of the leaves on the formation of fractal dimensions (D), was observed.

**TABLE 3. Matrix table with correlations**

	L	w	SLA	Per	D
L		3.26E-67	8.35E-61	2.01E-72	1.12E-33
w	0.973		5.79E-73	3.17E-82	1.20E-35
SLA	0.964	0.979		5.03E-79	1.35E-41
Per	0.978	0.986	0.984		2.24E-36
D	0.872	0.883	0.912	0.887	

$$D = 1.73756 + 0.001117 \cdot L - 0.00158 \cdot W - 0.00012 \cdot Per + 0.001677 \cdot SLA \quad (6)$$

By multiple regression analysis, equations were obtained that described the variation of the fractal dimension D according to L and W, equation (7), respectively according to W and Per, equation (8), under high statistical safety conditions. The values of the statistical safety coefficients for equations (7) and (8) are shown in table 4.

Based on the two equations (7) and (8), 3D graphical distributions were obtained, which express the fractal dimension (D) variation in relation to considered parameters of the leaves. Also, graphical representations were obtained in the form of isoquants, which express graphically the optimal range of x and y variation for optimal values of fractal dimensions (D). The 3D and isoquant graphical representations for equation (7), as example, are presented in Figure 2 a, b.

When the leaf surface (SLA) with each leaf parameter (L, W, Per) was

considered together, the resulting equations led to obtaining the fractal value D with higher accuracy, appreciation based on the p values. The values of the F test indicated the best description of the variation of the D value according to the perimeter (Per) and the leaf surface (SLA):  $F = 269016.2$  in the case  $D = f(\text{Per}, \text{SLA})$ , equation (9), figure 3 a, b;  $F = 103204$  in the case of  $D = f(L, \text{SLA})$ , and  $F = 69267.15$  in the case of  $D = f(W, \text{SLA})$  respectively.

$$D = ax^2 + by^2 + cx + dy + exy + f \quad (7)$$

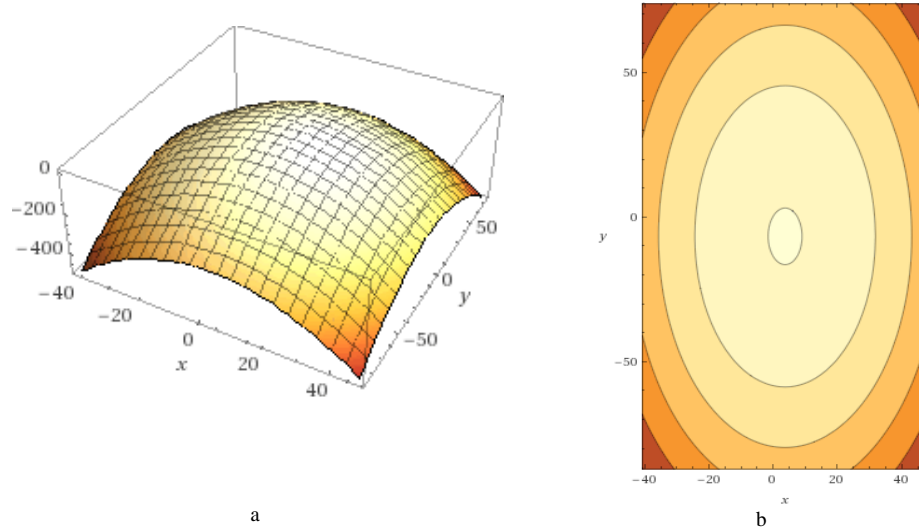
where: x – L – leaf length;  
y – W – leaf width;  
a, b, c, d, e, f - the equation (7) coefficients;  
a= -0.131015323791136;  
b= -0.0382441232157529;  
c= 0.969826215291039;  
d= -0.513763983011542;  
e= 0.142354323387578;  
f= 0.

$$D = ax^2 + by^2 + cx + dy + exy + f \quad (8)$$

where: x – W – leaf width;  
y – Per – leaf perimeter;  
a, b, c, d, e, f - the equation (8) coefficients;  
a= -0.0340246299945366;  
b= -0.000163168952235456;  
c= -0.933039520684597;  
d= 0.0402541620015277;  
e= 0.00585110570318677;  
f= 0.

**TABLE 4. Statistical safety parameters of the (7) and (8) equations**

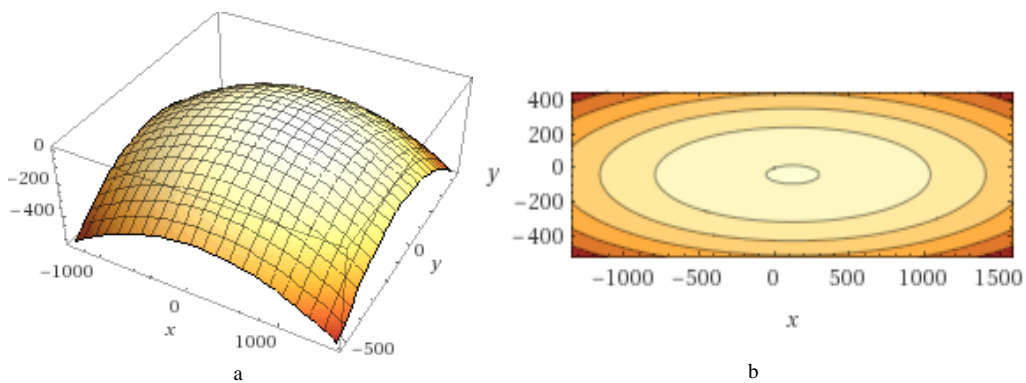
Equation parameter	Equation (7)			Equation (8)		
	R <sup>2</sup>	F	P-value	R <sup>2</sup>	F	P-value
x <sup>2</sup>			0.000672			0.429747
y <sup>2</sup>			0.32631			7.64E-06
x	0.995	8175.983	6.9E-24	0.994	7039.715	1.68E-12
y			7.79E-08			6.79E-25
xy			0.064621			0.016813
Intercept			-			-



**FIGURE 2.** Graphical distribution of fractal dimension (D) in relation to L (x) and W (y);  
 a – 3D graphic distribution; b – distribution in the form of isoquants

$$D = ax^2 + by^2 + cx + dy + exy + f \quad (9)$$

where: x – Per – leaf perimeter;  
 y – SLA – scanned leaf area;  
 a, b, c, d, e, f - the equation (9) coefficients;  
 a= -0.000124311614807094;  
 b= -0.00133565634546919;  
 c= 0.0303471921735137;  
 d= -0.108231438569611;  
 e= 0.000839994428690411;  
 f= 0.



**FIGURE 3.** Graphical distribution of fractal dimension (D) in relation to L (x) and W (y);  
 a – 3D graphic distribution; b – distribution in the form of isoquants

From the results of multiple regression analysis, the conditions in which each independent foliar parameter can assess fractal dimension D were analyzed. Based on leaves lengths (L), relation (10) describes the variation of fractal dimension (D), with  $R^2=0.964$ ,  $RMSEP = 0.10801$ ,  $SEP = 0.612707$ .

$$D = 0.0014 \cdot L^2 - 0.0084 \cdot L + 1.7514 \quad (10)$$

Based on leaves widths (W), relation (11) describes the variation of fractal dimension (D), with  $R^2=0.973$ ,  $RMSEP = 0.01068$ ,  $SEP = 0.606469$ .

$$D = 0.0014 \cdot L^2 - 0.0084 \cdot L + 1.7514 \quad (11)$$

Based on leaves perimeters (Per), relation (12) describes fractal dimension (D) variation, with  $R^2=0.986$ ,  $RMSEP = 0.010367$ ,  $SEP = 0.588325$ .

$$D = 9E-07 \cdot Per^2 - 0.0001 \cdot Per + 1.7362 \quad (12)$$

Based on leaves scanned area (SLA), relation (13) describes the variation of fractal dimension (D) with de  $R^2=0.998$ ,  $RMSEP = 0.010107$ ,  $SEP = 0.573504$ .

$$D = 3E-06 \cdot SLA^2 + 0.001 \cdot SLA + 1.7266 \quad (13)$$

The leaves are organs that faithfully express the relationship of plants with general environmental conditions, with the nutrition media, stress factors, vegetation stages, production and production quality (Jivan & Sala, 2014; Rawashdeh & Sala, 2015, 2016; Datcu & Sala, 2018). At the same time, the leaves of some species of plants, considered as indicator species, have been used in biomonitoring studies in urban or non-urban habitats (Ianovici, 2011; Ianovici *et al.*, 2015; Datcu *et al.*, 2017; Hajizadeh *et al.*, 2019). Non-invasive analysis is very useful in leaf studies, and different applications have been developed for this (Drienovsky *et al.*, 2017a,b).

Fractal analysis is a very useful method for studying shapes and surfaces (Xu *et al.*, 2009). Fractal analysis has been applied to plants in different studies to identify species, taxonomic studies, pomology studies etc. (Jobin *et al.*, 2012; Du *et al.*, 2013; Gazda, 2013; Bayirli *et al.*, 2014; da Silva *et al.*, 2015; Sala *et al.*, 2017).

Starting from the objectives of this study, a series of causality and interdependence relationships between the foliar parameters studied (L, W, Per, SLA) and the fractal dimension (D) were found. The fractal geometry of the leaves, through the defining elements analyzed, was faithfully highlighted in the fractal dimensions D. The causal relationships between the L, W, Per and SLA sheet parameters were identified under high statistical safety conditions.



## CONCLUSIONS

This study evaluated fractal dimension of black alder leaves. This index, which can be determined quickly and cheaply, is suitable in biomonitoring studies in urban or non-urban habitats. This species often occurs near rivers, but it can be found in parks in towns and cities. Fractal dimension can be analyzed in relation with various morphometric parameters, as perimeter, scanned leaf area, leaf length and width. In this study, mathematical models between these indices and fractal dimension were developed, statistical safety being assured. Fractal dimension D can be a stable index for the leaves geometry characterization, having a low degree of variability when compared with L, W, Per or SLA, which presented a high degree of variability, noticed based on variation coefficient CV.

## REFERENCES

- Ackerly D.D., Cornwell W.K. 2007. A trait-based approach to community assembly: partitioning of species trait values into within- and among community components. *Ecol. Lett.* 10: 135-145.
- Aniszewska M., Tulska E., Żurawska K. 2019. Variability of cone parameters and scale morphology in the black alder (*Alnus glutinosa* L.) in the context of seed extraction. *Eur. J. For. Res.* 138: 981-989.
- Bayirli M., Selvi S., Cakilcioglu U. 2014. Determining different plant leaves' fractal dimensions: a new approach to taxonomical study of plants. *Bangl. J. Bot.* 43(3): 267-275.
- Buczkowski S., Kyriacos S., Nekka F., Cartilier L. 1998. The modified box-counting method: analysis of some characteristic parameters. *Pattern Recognit.* 31(4): 411-418.
- Costa L.F., Cesar L.F. 2000. Shape analysis and classification: Theory and Practice CRC Press, Pennsylvania, 680 pp.
- Da Silva N.R., Florindo J.B., Gómez M.C., Rossatto D.R., Kolb R.M., Bruno O.M. 2015. Plant identification based on leaf midrib cross-section images using fractal descriptors. *PLoS ONE* 10(6): e0130014.
- Datcu A.-D., Sala F. 2018. Studies regarding the influence of nitrogen fertilizing dose on some ecophysiological parameters for *Triticum aestivum*. *Res. J. Agric. Sci.* 50(4): 105-110.
- Datcu A.-D., Sala F., Ianovici N. 2017. Studies regarding some morphometric and biomass allocation parameters in the urban habitat on *Plantago major*. *Res. J. Agric. Sci.* 49(4): 96-102.
- Díaz S., Lavorel S., de Bello F., Quétier F., Grigulis K., Robson T.M. 2007. Incorporating plant functional diversity effects in ecosystem service assessments. *Proc. Nat. Acad. Sci. USA* 104: 20684-20689.
- Drienovsky R., Nicolin A.L., Rujescu C., Sala F. 2017a. Scan LeafArea – A software application used in the determination of the foliar surface of plants. *Res. J. Agric. Sci.* 49(4): 215-224.
- Drienovsky R., Nicolin A.L., Rujescu C., Sala F., 2017b. Scan Sick & Healthy Leaf – A software application for the determination of the degree of the leaves attack. *Res. J. Agric. Sci.* 49(4): 225-233.
- Du J.-x., Zhai C.-M., Wang Q.-P. 2013. ecognition of plant leaf image based on fractal dimension features. *Neurocomputing* 116: 150-156.
- Ellenberg H. 2009. Vegetation ecology of Central Europe. Cambridge University Press. 4<sup>th</sup> Ed., 756 pp.
- Gazda A. 2013. Fractal analysis of leaves: are all leaves self-similar along the cane? *Ekológia (Bratislava)* 32(1): 104-110.
- Gosling P.G., McCartan S.A., Peace A.J. 2009. Seed dormancy and germination characteristics of common alder (*Alnus glutinosa* L.) indicate some potential to adapt to climate change in Britain. *Forestry: An International Journal of Forest Research* 82(5): 573-582.
- Hajizadeh Y., Mokhtari M., Faraji M., Abdolahnejad A., Mohammadi A. 2019. Biomonitoring of airborne metals using tree leaves: Protocol for biomonitor selection and spatial trend. *MethodsX* 6: 1694-1700.
- Hammer Ø., Harper D.A.T., Ryan P.D. 2001. PAST: Paleontological Statistics software package for education and data analysis. *Paleontol. Electron.* 4(1): 1-9.
- Han J.-H., Jeong J.-H., Lee G.-Y., Kim B.-R. 2015. Studies on wood quality and growth of *Alnus glutinosa* (L.) Gaertn. in Korea. *J. Korean Wood Sci. Technol.* 43(1): 1-8.
- Houston Durrant T., de Rigo D., Caudullo G., 2016. *Alnus glutinosa* in Europe: distribution, habitat, usage and

**SALA et al:** Fractal dimension and causality relationships with foliar parameters: case study at *Alnus glutinosa* (L.) Gaertn.

- threats. In: San-Miguel-Ayanz J., de Rigo D., Caudullo G., Houston Durrant T., Mauri A. (Eds.), European Atlas of Forest Tree Species. Publ. Off. EU, Luxembourg, pp. 64-65.
- Ianovici N., Vereş M., Catrina R.G., Pîrvulescu A.-M., Tănase R.M., Datcu D.A. 2015. Methods of biomonitoring in urban environment: leaf area and fractal dimension. *Annals of West University of Timișoara, ser. Biology XVIII* (2): 169-178.
  - Ianovici N. 2011. Approaches on the invasive alien taxa in Romania - *Ambrosia artemisiifolia* (ragweed) II, *Annals of West University of Timișoara, ser. Biology*, 14: 93-112
  - Jivan C., Sala F. 2014. Relationship between tree nutritional status and apple quality. *Hort. Sci.* 41(1): 1-9.
  - Jobin A., Nair M.S., Tatavarti R. 2012. Plant identification based on Fractal Refinement Technique (FRT). *Proc. Technol.* 6: 171-179.
  - Kaliniewicz Z., Markowski P., Anders A., Kaliniewicz Z., Markowski P., Anders A., Jadwisieńczyk B., Poznański A. 2018. Correlations between germination capacity and selected properties of black alder (*Alnus glutinosa* Gaertn.) achenes. *Baltic Forestry* 24(1): 68-76.
  - Kaliniewicz Z., Trojanowski A. 2011. Variability analysis and correlation of selected physical properties of black alder seeds. *Inżynieria Rolnicza* 8(133): 167-172.
  - MacLeod N., Steart D. 2015. Automated leaf physiognomic character identification from digital images. *Paleobiology* 41(4): 528-553.
  - Mandelbrot B.B. 1977. *Fractals: Form, Chance, and Dimension*, W.H. Freeman, San Francisco, 365 pp.
  - McGill B.J., Enquist B.J., Weiher E., Westoby M. 2006. Rebuilding community ecology from functional traits. *Trends Ecol. Evol.* 21: 178-185.
  - Mitchell A.F. 1974. *A field guide to the trees of Britain and northern Europe*, Collins Publisher, 415 pp.
  - Rasband W.S. 1997. *ImageJ*. U. S. National Institutes of Health, Bethesda, Maryland, USA, pp. 1997-2014.
  - Rawashdeh H.M., Sala F. 2016. Effect of iron and boron foliar fertilization on yield and yield components of wheat. *Rom. Agric. Res.* 33: 241-249.
  - Rawashdeh H.M., Sala F. 2015. Effect of some micronutrients on growth and yield of wheat and its leaves and grain content of iron and boron. *Bulletin UASVM series Agriculture* 72(2): 503-508.
  - Rutkowski P., Konatowska M., Wajsowicz T.S. 2019. Tree-soil-water relationships in European black alder forest - Case study. *Mechanization in Agriculture & Conserving of the Resources* 65(6): 200-203.
  - Sala F., Iordănescu O., Dobrei A. 2017. Fractal analysis as a tool for pomology studies: Case study in apple. *AgroLife Sci. J.* 6(1): 224-233.
  - Sala F., Arsene G.-G., Iordănescu O., Boldea M. 2015. Leaf area constant model in optimizing foliar area measurement in plants: A case study in apple tree. *Sci. Horticulture (Amsterdam)* 193: 218-224.
  - Socha J., Ochał W. 2017. Dynamic site index model and trends in changes of site productivity for *Alnus glutinosa* (L.) Gaertn. in southern Poland. *Dendrobiology* 77: 45-57.
  - Tricot C. 1995. *Curves and Fractal Dimension* Springer-Verlag, New York, 324 pp.
  - Vacek Z., Vacek S., Podrázský V., Král J., Bulušek D., Putalová T., Baláš M., Kalousková I., Schwarz O. 2016. Structural diversity and production of alder stands on former agricultural land at high altitudes. *Dendrobiology* 75: 31-44.
  - Voss R. 1985. Random fractal forgeries. In: Earnshaw R. (Ed.) *Fundamental algorithms for computer graphics*, Springer Verlag, Berlin, pp. 805-835.
  - Xu Y., Ji H., Fermüller C. 2009. Viewpoint invariant texture description using fractal analysis. *Int J. Comput. Vis.* 83: 85-100.
  - Yaman B. 2009. Wood anatomy of ivy-hosting black alder (*Alnus glutinosa* Gaertn.). *Dendrobiology* 62: 41-45.